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SPATIAL CORRELATION IN DISCRETE MARKOV RANDOM FIELDS AND TRUNCATED GAUSSIAN PROCESSES

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Abstract. *The characterization of physical properties of petroleum reservoirs is the initial point in the forecasting of the fluids flow behavior in porous media. In this subject, the simulation of stochastic processes has an important role, by generating images of the variable, with specific characteristics of spatial correlation and continuity. In this work we discuss relevant aspects concerning the use of discrete Markov Random Fields and Gaussian Stochastic Processes in the representation of rock properties in fluids reservoirs. The main point focused is the relationship between the spatial correlation in Gaussian processes and the attraction parameter in Markov random fields, here studied by measuring the autocorrelation parameters in binary Markov images, generated by the Metropolis algorithm. Gaussian images with Gaussian type autocorrelation, after truncated in binary facies, has a correspondence to Markov images. This similarity is validated by analysis in the autocorrelation function of the discrete Gaussian processes.*

Keywords*: Markov Random Fields, Gaussian Stochastic Processes, Metropolis algorithm, Autocorrelation*

I. INTRODUCTION

Markov and Gaussian stochastic processes belong to the class termed point-based or pixel-based process, that consist on the simulation of the values of the variable in each pixel of the image. The variable focused in this work is the reservoir rock facies, represented in the network by the different colors of the image.

In the use of Gaussian processes, one must assume conditions of stationarity and ergodicity to generate random function with an expected autocorrelation function. Several methods are used to simulate Gaussian process, extensively used in geostatistical studies, among which it may be cited the Turning Bands method, proposed by Montoglou and Wilson (1982), the covariance matrix decomposition, presented in Davis (1987a,1987b) e Alabert (1987), and the Sequential Gaussian Simulation, proposed by Journel and Alabert (1990). The simulation of discrete variables is done by the simulation of the continuous variable, followed by a truncation of this variable, according to a specific proportion between the facies or colors. This approach is denominated Truncated Gaussian Simulation, and may be viewed in Journel and Isaaks (1984), Journel and Posa (1990), and Galli et alii (1994) .

In the application of Markov Random Fields (MRF), the variable in a site is only related to the points of its neighborhood. They are simulated in a sequential approach that consists on successive exchanges in the image of the variable, according to a transition probability, that is related to the expected value of the attraction parameter β. Most algorithms used to simulate MRF are based in the work of Metropolis et alii (1953), and may be applied to image restoration, as presented by Geman and Geman (1984), and texture analysis, as presented in Flinn (1974) and Cross and Jain (1984). The use of colored lattices may be practical to describe images of discrete variables with more than two possible facies, and is proposed by Strauss (1975 and 1977).

The autocorrelation function in Gaussian processes and the attraction parameter in Markov random fields are characteristics of continuity of the system. Starting from this motivation, the second goal of this work is to research the relationship between these parameters, and, as a consequence, the relationship and similarity between the images generated from Markov and Gaussian processes. This purpose is achieved by measuring the parameters of spatial correlation observed in the images of MRF, and relating to a theoretical autocorrelation model. Visual comparison between the images of both process and the study of the spatial correlation of truncated Gaussian processes is helpful to explain the observed similarities.

This paper is organized as follows. The fundamental aspects of Markov random fields are described in next section and the simulation of binary Markov processes are presented in Section III. In Section IV, we discuss the practical results of the experiments evaluating the spatial correlation in binary Markov images, and study the relationships between Markov and Gaussian images. Section V contains the concluding remarks.

II. MARKOV RANDOM PROCESSES

The Markov chains describe temporal sequences of random variables, governed by transition probabilities. The system "state" is represented by the value of the random variable. Markov Random Fields (MRF) may be viewed as an extension of the Markov chains from random variables to stochastic processes. In this case, the system "state" is represented by the spatial image of the variable in an iterative sequence.

The Markov property, defined for temporal processes, is expanded for spatial processes, and the stochastic Markov process is determined by the conditional probabilities at each point with respect to the points of its spatial neighborhood. There are several configurations to define this neighborhood, the most common being that of the four nearest neighbors. A process is said to be a MRF if, for the neighborhood ∂_s of each pixel s, the following relationship holds:

$$p\left(X_s = x_s \middle| X_r = x_r, r \neq s\right) = p\left(X_s = x_s \middle| X_r = x_r, r \in \partial_s\right) \tag{1}$$

Basing on the analysis of Spitzer (1971) and Geman & Geman (1884), one verifies that a process is a MRF with respect to a neighborhood if and only if its joint probability is a Gibbs distribution. For the known Ising model (binary MRF), the local conditional probability is calculated as a function of the points of neighborhood (v_i) and of the attraction parameter β:

$$p\left(X_i = x_i \middle| X_r = x_r, r \neq s\right) = \frac{exp\left(-\beta \cdot x_i \sum v_i\right)}{1 + exp\left(-\beta \sum v_i\right)} \tag{2}$$

The relationship between the probabilities of two configurations X e X', which allows to establish the exchange probability, can be calculated by the equation, proposed by Besag (1974):

$$\frac{P(X)}{P(X')} = \prod_{i=1}^{n} \frac{P\left(x_i \middle| x_1,\ldots,x_{0-1}, x'_{i+1},\ldots,x'_n\right)}{P\left(x'_i \middle| x_1,\ldots,x_{i-1}, x'_{i+1},\ldots,x'_n\right)} \qquad (2)$$

This equation is rather practical and simple to implement, being helpful in algorithms of MRF simulation and image restoration.

III. SIMULATION OF MARKOV RANDOM FIELDS

The simulation of discrete Markov processes starts from a random image upon which one changes values at one point (single-flip) or at a couple of points (spin- exchange), according to the transition probability between the initial and the modified images. This sequence is an extension of the Markov chains for stochastic processes, and the transition probability is related to the joint probability to be attained by the end of the sequence.

The Metropolis algorithm, proposed by Metropolis alii (1953), is the basis for the most sequential algorithms, and consists on a sequence of exchanges, governed by the Gibbs distribution previously presented. The stabilization at the joint specified probability, as well as the process ergodicity, is assured in the work of Geman & Geman (1984).

The method can be used to simulate reservoir rocks, by making the assumption that the values 0 and 1 (black and white) represent the reservoir sandstone and shale (permeable and non permeable facies). The proportion between the facies is supposed to be known, and is kept constant. Then, the passing from one image to other is done through double exchanges between two random points in the network.

The exchange sequence stop when the global energy achieves a plateau; another valid criterion is the stabilization of the rate of exchange (number of exchanges per iteration). Frery (1991) presents another stopping rule, which uses the sequential estimate of the attraction parameter β; the sequence stop when the image surpasses the previously established value of β.

Figure 1 shows the stabilization of the number of exchanges and of the attraction parameter in relation to the number of iterations, for a value of the attraction β equal to 0.9. Each iteration corresponds to a number of exchange trials equal to the number of points in the network, that is, it is equivalent in quantity to a scan-type passage through the image.

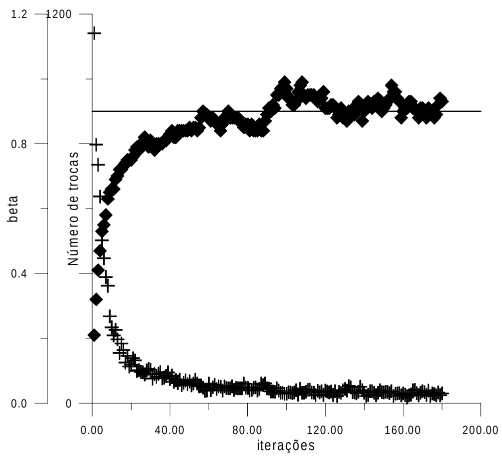

Figure 1 – Stabilization of the number of exchanges and of the attraction value, in the Metropolis algorithm

The algorithm named *Gibbs Sampler*, proposed in Geman & Geman (1984), consists on scanning the image, and sampling, pixel by pixel, a value for the variable related to its conditional probability. It applies both to discrete and continuous variables, and it is useful in Bayesian approaches, to simulate MRF subjected to complex conditions as degraded image restoration, used in Geman e Geman (1884), inequalityies conditions, used in Freulon and de Fouquet (1993), connectivity conditions used in Allard (1994), and many others.

In present work, the images of the binary variables are generated according to the Metropolis algorithm, and the sequence stop by the criterion of Frery (1991). Figure 2 shows an example of the sequential approach. The first image on the top is the initial image generated at random with proportion 50% between the colors; in the same figure it may be observed the images after 10 iterations (at the middle), and after 50 iterations (at the bottom).

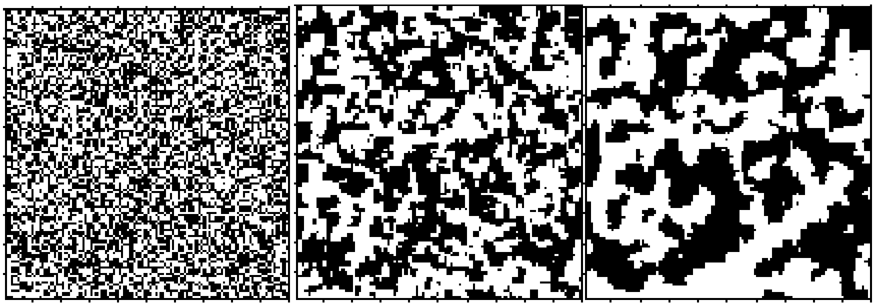

Figure 2 – Sequential simulation of images in the Metropolis algorithm – initial image (top), intermediate image after 10 iterations (middle) and final image after 50 iterations (base)

Some resulting images generated by this sequence are presented in Figure 3, for several values of β, in networks of 160 x 160 pixels. It can be seen that the grater the values of β, the greater the size of the clusters.

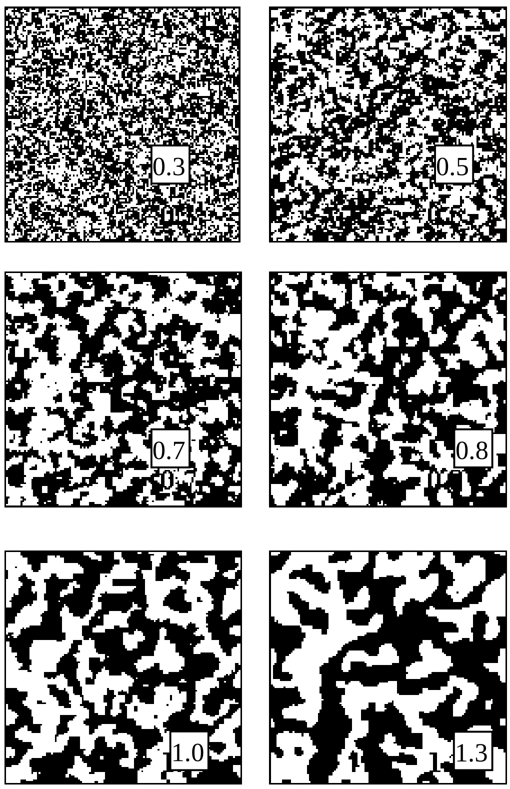

Figure 3 –Images with 160x160 pixels, simulated with Metropolis algorithm for values of β in the figure

IV. PARAMETERS OF SPATIAL CORRELATION IN MARKOV PROCESSES

The simulation of images in Gaussian processes is based on the knowledge of a second order statistical parameter (autocorrelation, autocovariance, or semivariogram). In geostatistics it is frequent the use of the semivariogram function $\gamma(h)$ to express the spatial correlation. In stationary stochastic process, this function is related to autocovariance $C(h)$ as:

$$\gamma(h) = 1 - C(h)$$

Current models for the semivariogram are:

Gaussian model:

$$\gamma(h) = 1 - exp\left(-3h^2/a^2\right)$$

Isotropic exponential model:

$$\gamma(h) = 1 - exp\left(-3h/a\right)$$

Factorized exponential model:

$$\gamma(h) = 1 - exp\left(-3h_x/a - 3h_y/a\right)$$

where h is the isotropic distance between two pixels, h_x and h_y are the distances in the main directions x and y, and a is the range of the correlation.

To improve the comprehension about the properties of Markov processes, the values of the semivariogram function were calculated for the images generated with the Metropolis algorithm.

For different values of the attraction parameter β, 100 images with dimensions 160x160 pixels were generated, from which the semivariogram values were calculated at the various distances (in pixels) in the two main directions. The average values were adjusted to a known model.

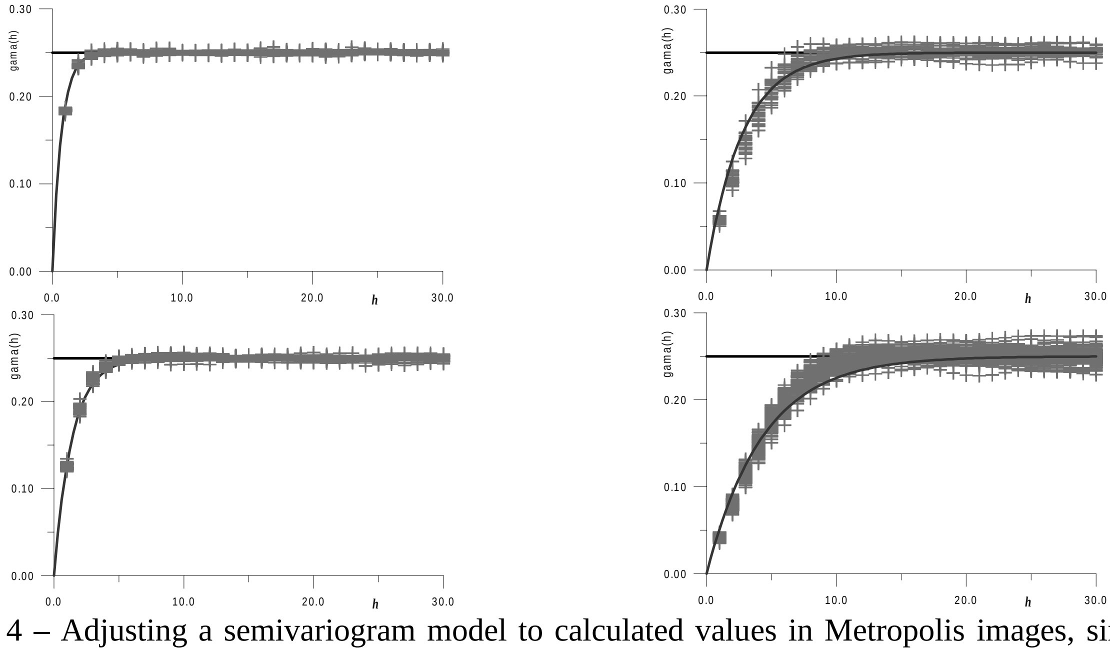

Figure 4 – Adjusting a semivariogram model to calculated values in Metropolis images, simulated with β equal to 0.3, 0.5, 0.8 and 1.0.

Figure 4 shows the adjust for β=*0.3, 0.5, 0.8* and *1.0*. The exponential model gives the best adjust and the greater values of β correspond to the greater values of the parameter *a*.

Although the semivariograms are similar on both directions, indicating an isotropic characteristic, it would be possible that the factorized exponential model were also valid, since it presents equal values on the main directions. For this reason, the semivariogram of the generated images was investigated in the 45^o and 135^o directions. Results show rather close values of range in all directions, indicating the existence of an isotropic exponential model.

Thus, the images of discrete Markov processes, defined with four-point neighborhood, and generated by the Metropolis algorithm with double exchange, present an isotropic exponential model of spatial correlation.

A empirical relationships between the parameter β and the semivariogram for the distance of one pixel $\gamma(1)$ is:

$$\gamma(1).\beta = 0{,}05$$

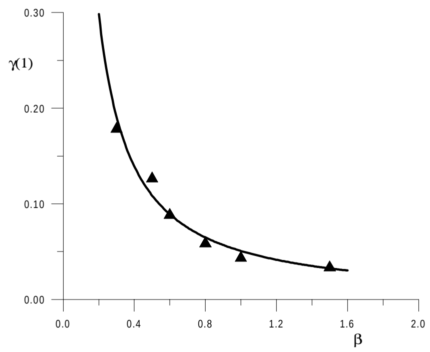

Figure 5 – Relationship between the attraction β and the semivariogram for the distance of one pixel γ(1) – observed values and adjusted curve

This relationship is shown in Figure 5 and may be helpful to achieve a practical relationship between β and the range *a* of the spatial correlation.

Assuming that the semivariogram is exponential, according to the experimental data, we have:

$$\gamma(1) = 0{,}25[1 - exp(-3/a)]$$

Thus, the range *a* of the spatial correlation can be related to β as:

$$a = \frac{3}{ln\left(\dfrac{\beta}{\beta - 0{,}20}\right)}$$

This equation is represented in Figure 6, and provides a good agreement with the observed values.

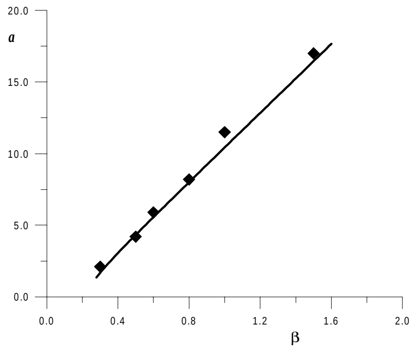

Figure 6 – Relationship between parameter β and the range of the semivariogram observed in images - observed values and empirical relationship

From these analyses, one could conclude that images generated by binary Markov processes would be similar to those generated by Gaussian processes simulated with exponential type autocorrelation model. But it is not correct. To go deeper into the problem, it is worthwhile to observe images simulated by both processes. In Figure 7, it is presented a Metropolis image with β=0,8 and Gaussian images with different correlation models.

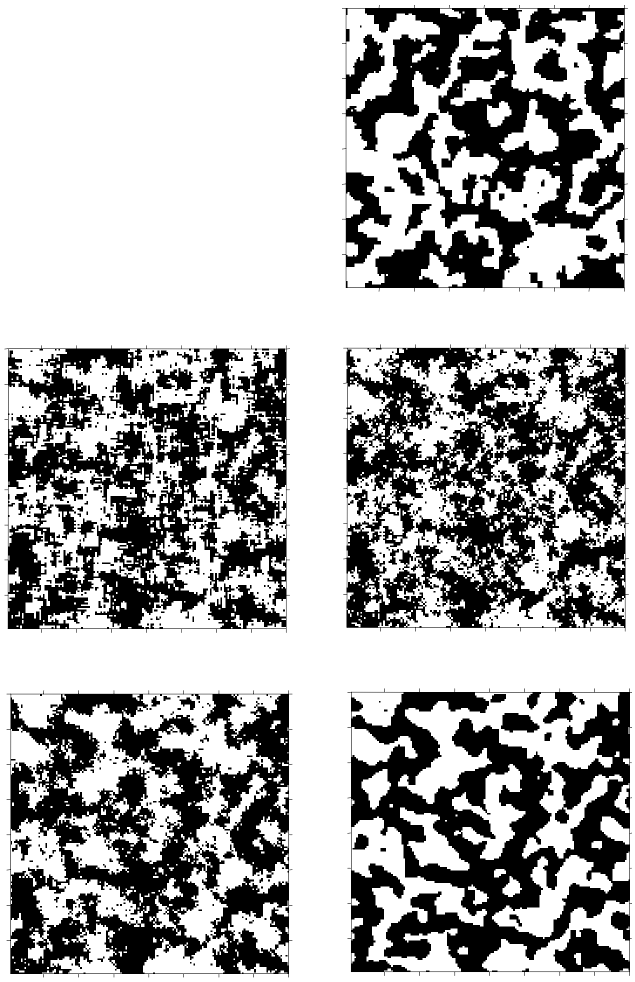
Figure 7 –Metropolis image (on the top) compared to Gaussian images with factorized exponential correlation model (left at the middle), isotropic exponential model (right at the middle), spherical model (left at the bottom) and Gaussian model (right at the bottom)

It is evident that the Markovian image is visually similar to the Gaussian autocorrelation image. This visual conjecture may be explained by the fact that the correlation model refers to the continuous variable before the truncation. The relation between the semivariograms of the continuous and discrete variable is theoretically demonstrated by Matheron (1989), and describe the semivariogram of the truncated variable as the square root of those of the continuous variable.

This behavior of the truncated variable is validated here by simulation of Gaussian process with Gaussian correlation. The average semivariogram calculated from 100 images may be observed in Figure 8, compared to the theoretical models.

The continuous variable with a Gaussian semivariogram (parabolic close to the origin) creates, after the truncation, a discrete variable with a linear semivariogram close to the origin (exponential). Thus, the images with similar visual aspect have also similar spatial correlation.

Thus, there is a correspondence between the geometric characteristics of Metropolis images and those of Gaussian processes, generated with a Gaussian correlation model to the continuous variable.

This relationship is also verified in Salomão (1998) by analyses of the flow properties, evaluated through the parameters of the Percolation Theory, in spatially correlated process. The percolation parameters, as the percolation threshold, are equivalent in Markov and Gaussian process with such characteristics.

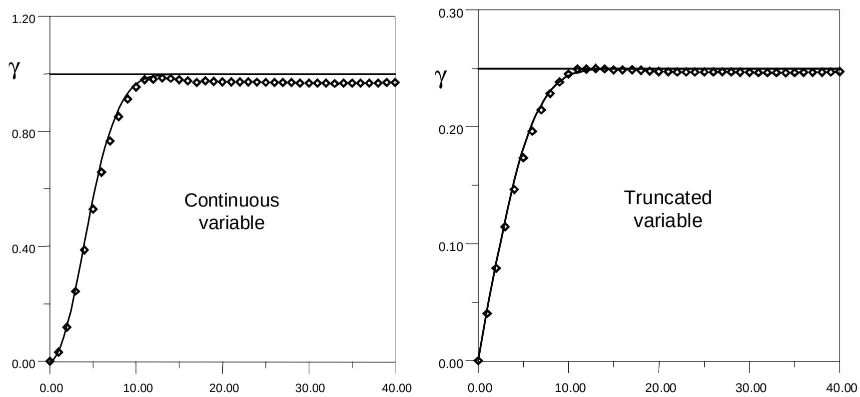

Figure 8 – Average semivariograms observed for continuous variable and for truncated variable, simulated by the Sequential Gaussian Simulation

V. CONCLUDING REMARKS

The attraction parameter of the binary Markov process may be related to the autocorrelation parameter of the Gaussian processes since both describes the continuity of the system. The binary Markov images are strictly similar to the truncated Gaussian images, whose continuous variable is generated with Gaussian type autocorrelation function. These results may be justified by the behavior of the spatial correlation of the truncated variable, and are corroborated by experiments relating the flow in porous media, in stochastic images, applying the Percolation theory.

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